Three-player game-theoretic model over a freight transportation network

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Abstract

With recent development in freight transportation industry, its network structure has become more complicated, as many decision-makers competing for profits with each other are involved. While most recent research in this area is focused on the perfectly competitive market and the prices are given as a constant tariff rate, little attention has been paid to the system optimization problem in the absence of regulatory authority. In this paper, we investigate the competitive equilibrium in an oligopolistic market on a freight network. A partially non-cooperative game among shippers, carriers and infrastructure companies (IC) is examined. All three kinds of players act as profit maximizing agents, except that the carriers and ICs are assumed to behave cooperatively in their own coalitions. We consider the vertically efficient non-linear tariff schedules which are commonly used in the transportation industry. By introducing a three-stage game-theoretic model, we show that the equilibrium flows can also maximize total system profits if the IC and the carrier both use vertically efficient nonlinear pricing schedules. The division of the surplus associated with each shipment is obtained by solving a linear programming problem. We provide a few examples under different situations to show the existence of the resulting equilibrium.

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1. Introduction

The Freight Network Equilibrium Problem (FNEP) has attracted a lot of attention over the past two decades, but most previous studies involve two agents only: the shipper and the carrier. Typically, a simultaneous shipper–carrier equilibrium for the FNEP can be formulated in the following way: the shipper, or so called the manufacturer, controls a set of origin-to-destination shipments, with the demand for each shipment decided by an inverse demand function. To get the shipments to the market, the shipper must use a carrier or coalition of carriers who control not only the conveyance but also the transportation infrastructure. The shipper selects an output while the carriers charge shipper a freight rate obtained by maximizing their own profits. With such
a setting, a game theoretical approach can be applied to formulate and analyze FNEP. In general, two kinds of alternative objectives are used to describe the agents’ behavior: User Equilibrium (UE) or System Optimum (SO) (Harker, 1987, 1988). In Hurley and Petersen (1994), UE principle is used to model shipper’s behavior and the carrier is assumed to behave according to the SO principle; by using a particular form of nonlinear tariff, they showed that the UE and SO can be simultaneously satisfied in an incomplete market. Very recently, game-theoretical approaches were also employed to investigate other aspects of the FNEP. Bassanini et al. (2002) examined railroad capacity allocation and track time pricing; Castelli et al. (2004) considered a strategic game between two players acting on the same road transportation network, where the first player aims at minimizing transportation costs, whereas the second player aims at maximizing profit.

However, in recent years the freight industry throughout the world is in a process of restructuring and commercialization. Regulations are designed to liberalize the market for providing transportation service and enhance competition within the sector. The restructuring has normally involved the separation of the infrastructure from operations. People expect that this would result in both the rational distribution of the resources and increased efficiency in service provision. In general, the freight industry in the future can be considered to comprise three types of agents: the shipper, the carrier and the infrastructure company (IC). When the system involves more than two players in a hierarchical game, the computation of equilibrium is incomparably more difficult (Basar and Olsder, 1999). To simplify the problem, Nagurney (1999) and Nagurney et al. (2001) used a fully decentralized model to formulate and solve the FNEP using a variational inequality approach. Nevertheless, when different players are in asymmetric positions, their models are difficult to analyze the interactions among the players with different objective functions, and with the model the planner can hardly manage with the goal of system optimization through appropriate form of tariff. The difficulty of such problems is that when the freight industry moves to a pure market economy, we must face the situation wherein each one of the players has the ability to enforce his or her decision on the other players who react independently and rationally.

Our approach, in contrast, provides a different point of view. We develop a competitive equilibrium model for three types of players (one shipper, multiple carriers and multiple ICs) in either symmetric or asymmetric positions, each acting as profit maximizing agents. In order to gain an SO solution, we restrict the carriers and ICs to two-part tariffs, which has a general form: \( R(x) = b + ax \), where \( x \) is the total shipment. In our analysis, under the oligopoly equilibria, parameter “\( a \)” will be set to the players’ marginal costs and parameter “\( b \)” affects a profit distribution between the players. Without loss of generality, we consider a three-stage sequential decision process. First, the ICs decide a tariff to the carriers, according to their own cost function and the information they have about the shipper and the carriers. Then, the carriers determine another tariff to the shipper, according to their own cost function, the tariff given by the ICs, and the information they have about the shipper. Finally, the shipper decides the quantity of the production to maximize its own profit. In each stage, we refer the agent who gives the tariff to as an “upper level” agent, and correspondingly, the agent who accepts the tariff as a “lower level” agent. On one side, the carriers and ICs have a choice of parameters, “\( a \)” and “\( b \)”, of the tariff. On the other side, when there are multiple carriers and ICs, the shipper can freely choose which carrier to transmit shipment and the carrier can freely choose the ICs. In terms of game theory, the problem of interest can be considered as a strengthening of the Nash equilibrium known as subgame perfect Nash equilibrium. The central idea underlying this concept is the principle of sequential rationality: equilibrium strategies should specify optimal behavior from any point in the game onward (Mas-Colell et al., 1995), which means, in this model, each agent will choose its own profit-maximizing strategy, given the “upper level” agents’ strategies and “lower level” agents’ information. Since it is a finite game of perfect information, we can thus apply the idea of backward induction: determining the optimal actions sequentially for moves at the final decisions at each stage. We show that in this game, by using the nonlinear tariff, the equilibrium of the Nash equilibrium is also the system optimum. Furthermore, the model predicts the set of tariff schedule of the carriers and ICs and provides a division of surplus among the players.

The paper is structured as follows. Section 2 contains a simple example with one shipper, one carrier and one IC. A three-player non-cooperative game between the three agents is analyzed step by step. The model results in a vertically efficient flow which is also an equilibrium. In Section 3, the example is extended to include competition among multiple ICs. We introduce an \( n \)-person cooperative cost-saving game, and show that the model leads to an equilibrium. In Section 4, we offer a more general equilibrium result by considering
a network of carriers and ICs and provide an economic explanation of the outcomes of the model. Section 5 concludes the paper with a brief summary.

2. One shipper, one carrier and one IC

We begin with a simple example by assuming that there is only one shipper with a single product, one carrier and one IC of track (or highway). The shipper’s product can be transmitted to the market by the sole carrier, and the carrier must use the track or highway held by the sole IC.

We can simply consider the shipper, the carrier and the IC as three players, numbered as player 1, 2 and 3. For convenience, we also define a function for each player:

\[ \pi_1(x) = xp(x) - c_1(x) - R(x) \]
\[ \pi_2(x) = R(x) - c_2(x) - S(x) \]
\[ \pi_3(x) = S(x) - c_3(x) \]

The total profit of the delivery market is

\[ \Gamma(x) = \pi_1(x) + \pi_2(x) + \pi_3(x) = xp(x) - c_1(x) - c_2(x) - c_3(x) \]

We assume that \( \Gamma(x) \) is concave with a maximum at \( x^* \). If both \( x^* > 0 \) and \( \Gamma(x^*) = \max \Gamma(x) > 0 \) hold, then it is economical to move the freight to the market. Here \( x^* \) is called the vertically efficient flow, and from the first-order optimality condition of \( \max \Gamma(x) \), the following relation obviously holds:

\[ \text{MR}_0(x^*) = \text{MC}_1(x^*) + \text{MC}_2(x^*) + \text{MC}_3(x^*) \]

where \( \text{MR}_0(x) \) is the marginal revenue, \( \text{MC}_1(x) \), \( \text{MC}_2(x) \) and \( \text{MC}_3(x) \) are separately the marginal costs of the shipper, carrier and IC. For convenience, we also define a function for each player:

\[ \mu_i(x) = x \left[ \text{MC}_i(x) - \frac{c_i(x)}{x} \right], \quad i = 1, 2, 3 \]

**Lemma 1.** Suppose \( S(x) \) is given, let

\[ T(x) = \pi_1(x) + \pi_2(x) = xp(x) - c_1(x) - c_2(x) - S(x) \]

and assume that \( T(x) \) is concave with a maximum at \( \bar{x} > 0 \)

\[ T(\bar{x}) = \max T(x) \]

Furthermore, suppose that the carrier offers a two-part tariff of the form \( R(x) = \bar{b} + \bar{a}x \) for \( x > 0 \), where

\[ \bar{a} = \text{MC}_2(\bar{x}) + \frac{dS(\bar{x})}{dx} \]

\[ \bar{b} = 2T(\bar{x}) - \mu_2(\bar{x}) - \bar{x} \left[ \frac{dS(\bar{x})}{dx} - \frac{S(\bar{x})}{\bar{x}} \right] \] for \( 0 \leq \bar{x} \leq 1 \)

Then the shipper will choose an output \( \bar{x} \), \( \pi_1 = (1 - \lambda)T(\bar{x}) \) and \( \pi_2 = \lambda T(\bar{x}) \). The parameter \( \lambda \) determines each agent’s share of the joint surplus. If \( \lambda = 1 \), the carrier receives all the surplus and the shipper receives none; if \( \lambda = 0 \), the shares are reversed.
Proof. If the carrier selects a two-part tariff of the form: \( R(x) = \bar{b} + ax \), then the shipper maximizes

\[
\pi_1(x) = xp(x) - c_1(x) - \left\{ \bar{b} + \left[ MC_2(\bar{x}) + \frac{dS(\bar{x})}{dx} \right] x \right\} \tag{11}
\]

The first-order condition for maximizing \( \pi_1(x) \) yields:

\[
MR_0(x) = MC_1(x) + MC_2(\bar{x}) + \frac{dS(\bar{x})}{dx} \tag{12}
\]

From the assumption that \( T(\bar{x}) \) is concave, clearly, \( x = \bar{x} \) is the unique and optimal solution to Eq. (12) as seen from the definitions (7) and (8), and is independent of the value selected for \( \bar{b} \), provided that \( \pi_1 \geq 0 \). Thus the shipper’s incentive is to produce an output \( \bar{x} \), and any other output would make the shipper worse off. Substituting \( \bar{b} \) in (10) into Eq. (11) yields

\[
\pi_1(\bar{x}) = \bar{x}p(\bar{x}) - c_1(\bar{x}) - \left\{ \lambda T(\bar{x}) - \mu_2(\bar{x}) - \bar{x} \left[ \frac{dS(\bar{x})}{dx} - \frac{S(\bar{x})}{\bar{x}} \right] + \left[ MC_2(\bar{x}) + \frac{dS(\bar{x})}{dx} \right] x \right\} = (1 - \lambda)T(\bar{x}) \tag{13}
\]

and

\[
\pi_2 = T(\bar{x}) - \pi_1 = \lambda T(\bar{x}) \tag{14}
\]

Since \( S(x) \) can be considered as part of the cost of the carrier, the rest of the proof is similar to that in Hurley and Petersen (1994) and is omitted here. □

From Lemma 1, given parameter \( \lambda \), we say that \( (a, b(\lambda)) \) is the equilibrium strategy of the carrier for the one shipper, one carrier game.

**Corollary 1.** With the assumptions in Lemma 1, for a given tariff, \( S(x) \), charged by the IC, if the carrier selects \( \lambda = 1 \), then the strategy \( (a, \bar{b}) \) and \( \bar{x} \) is an equilibrium for the one shipper, one carrier game, with \( \pi_1 = 0 \) and \( \pi_2 = T(\bar{x}) \).

**Proof.** By assumptions, the carrier is to maximize its profit with the limit that the profit of the shipper must be nonnegative. So the equilibrium must be reached at \( \lambda = 1 \). □

**Lemma 2.** Suppose that the IC offers a two-part tariff of the form \( b + ax \) for \( x > 0 \), where

\[
a = MC_3(x^*) \tag{15}
\]

\[
b = \lambda \Gamma(x^*) - \mu_3(x^*) \quad \text{for} \quad 0 \leq \lambda \leq 1 \tag{16}
\]

and furthermore, suppose that the carrier offers a two-part tariff of the form \( R(x) = \bar{b} + ax \) for \( x > 0 \). Then the shipper will choose an output \( \bar{x} = x^* \) with \( \pi_1 = (1 - \lambda)(1 - \lambda)\Gamma(x^*) \), \( \pi_2 = \lambda(1 - \lambda)\Gamma(x^*) \), and \( \pi_3 = \lambda \Gamma(x^*) \).

**Proof.** If the IC and the carrier both select two-part tariffs \( S(x) = b + ax \) and \( R(x) = \bar{b} + \bar{a}x \), then from Lemma 1

\[
\bar{T}(x) = \pi_1(x) + \pi_2(x) = xp(x) - c_1(x) - c_2(x) - b - MC_3(x^*)x \tag{17}
\]

Here, by definition (8) with \( \bar{T}(x) \) given by (7), \( \bar{x} \) must satisfy

\[
MR_0(\bar{x}) = MC_1(\bar{x}) + MC_2(\bar{x}) + MC_3(x^*) \tag{18}
\]

As we have assumed, \( \Gamma(x) \) is concave. In comparison with Eq. (5), \( \bar{x} = x^* \) is clearly the unique and optimal solution to Eq. (18). Thus, from Corollary 1 the carrier selects \( a = MC_2(x^*) + MC_3(x^*) \), the shipper maximizes

\[
\pi_1(x) = xp(x) - c_1(x)\{\bar{b} + [MC_2(x^*) + MC_3(x^*)]x\} \tag{19}
\]

The first-order optimality condition yields:

\[
MR_0(x) = MC_1(x) + MC_2(x^*) + MC_3(x^*) \tag{20}
\]
Thus the shipper’s incentive is to produce an output $x^*$. Substituting $\tilde{b}$ in Eq. (10) into (19) yields

$$
\pi_1 = x^*p(x^*) - c_1(x^*) - \left\{ \tilde{\lambda}[\Gamma(x^*) - S(x^*) + c_3(x^*)] - \mu_2(x^*) \right\} - x^*\left[MC_3(x^*) - \frac{S(x^*)}{x^*}\right] + [MC_2(x^*) + MC_3(x^*)]x^* = (1 - \tilde{\lambda})(1 - \tilde{\lambda})\Gamma(x^*)
$$

(21)

It is observed from Eqs. (13) and (14) that, regardless of the form of $S(x)$, $\pi_1$ and $\pi_2$ have the following relationship

$$
\frac{\pi_1}{\pi_2} = \frac{1 - \tilde{\lambda}}{\lambda}
$$

(22)

Substituting $\pi_1$ in (21) into (22) yields

$$
\pi_2 = \tilde{\lambda}(1 - \tilde{\lambda})\Gamma(x^*)
$$

(23)

$$
\pi_3 = \Gamma(x^*) - \pi_1 - \pi_2 = \tilde{\lambda}\Gamma(x^*)
$$

(24)

This completes the proof. □

From Lemma 2 we can see that parameters $\lambda$ and $\tilde{\lambda}$ together determine the agents’ shares of their joint surplus. In the first stage, the IC chooses parameter $\lambda$, which determines the shares of surplus between the IC and the rest surplus of the shipper and the carrier. In the second stage, parameter $\tilde{\lambda}$ is given by the carrier to determine the share of the rest surplus between the shipper and carrier. Since each player acts as a profit-maximizing agent, both IC and carrier will intend to select $\lambda$ and $\tilde{\lambda}$ as large as possible, respectively. Thus, we have the following result.

**Proposition 1.** If the shipper selects an output $x^*$, the IC selects $b = \Gamma(x^*) - \mu_2(x^*)$ and $a = MC_3(x^*)$, and the carrier selects $\tilde{b} = \Gamma(x^*) - \mu_2(x^*) - \mu_3(x^*)$ and $\tilde{a} = MC_2(x^*) + MC_3(x^*)$, then the strategy $(a, b, \tilde{a}, \tilde{b}, x^*)$ is an equilibrium for the game of one shipper, one carrier and one IC, with $\pi_1 = 0$, $\pi_2 = 0$ and $\pi_3 = \Gamma(x^*)$.

**Proof.** To show that $(a, b, \tilde{a}, \tilde{b}, x^*)$ is an equilibrium, it suffices to prove that no agent has an incentive to deviate from the equilibrium solution. This can be done by examining the following possible cases, respectively.

(i) If the IC selects $(a, b)$ and the carrier selects $(\tilde{a}, \tilde{b})$ as their tariff, then, by Lemma 2, the shipper will produce an output $x^*$ with $\pi_1 = 0$.

(ii) If the shipper selects $x^*$ and the carrier selects $(\tilde{a}, \tilde{b})$, then the IC solves the problem

$$
\max_{a, b} \pi_3 = b + ax^* - c_3(x^*), \quad \text{subject to } \pi_1, \pi_2 \geq 0
$$

(25)

Since $\lambda$ is arbitrarily chosen between 0 and 1, we substitute Eq. (16) into (25) and obtain

$$
\max_{a, \lambda} \pi_3 = \lambda\Gamma(x^*) + [a - MC_3(x^*)]x^*
$$

(26)

If $a \neq MC_3(x^*)$, then the total profit of all the three agents $\Gamma(x) < \Gamma(x^*)$ and $\pi_3 < \Gamma(x^*)$, which implies that the IC can further enhance its profit by choosing $a = MC_3(x^*)$. Thus, we must have $a = MC_3(x^*)$. Furthermore, the IC intends to maximize its profit by selecting an $\lambda$ as large as possible. If $\lambda > 1$, then $\pi_3 > \Gamma(x^*)$ and $\pi_1 + \pi_2 < 0$, which implies that either the shipper or the carrier will have a negative profit. Consequently, $\lambda = 1$ and $\pi_3 = \Gamma(x^*)$.

(iii) If the shipper selects $x^*$ and the IC selects $(a, b)$, then the carrier solves the problem

$$
\max_{\tilde{a}, \tilde{b}} \pi_2 = \tilde{b} + \tilde{a}x^* - c_2(x^*) - b - ax^*, \quad \text{subject to } \pi_1, \pi_3 \geq 0
$$

(27)

Since $\tilde{\lambda}$ is arbitrarily chosen between 0 and 1, from Eqs. (10) and (16), this is equivalent to

$$
\max_{\tilde{a}, \lambda} \pi_2 = (\tilde{\lambda} - 1)\Gamma(x^*) + [\tilde{a} - MC_2(x^*) - MC_3(x^*)]x^*
$$

(28)

From Corollary 1, the carrier can seize all the joint surplus of the shipper and the carrier. If the carrier chooses an $\tilde{a} \neq MC_2(x^*) + MC_3(x^*)$, then the total profit of both the shipper and carrier $\Gamma(x) < \Gamma(x^*)$, which implies
that the carrier can further enhance its profit by choosing \( \bar{a} = \text{MC}_2(x^*) + \text{MC}_3(x^*) \). Thus we must have \( \bar{a} = \text{MC}_2(x^*) + \text{MC}_3(x^*) \). Furthermore, the carrier intends to maximize its profit by selecting an \( \bar{\lambda} \) as large as possible. If the carrier selects \( \bar{\lambda} > 1 \), then, from Lemma 1, \( \pi_1 = (1 - \bar{\lambda})\bar{\Gamma}(\bar{x}) < 0 \). Therefore, the carrier can select \( \bar{\lambda} = 1 \) only and receive a profit \( \pi_2 = 0 \).

In summary, we conclude that Proposition 1 is true. \( \square \)

It is worth noting that there are no incentives to form a coalition among the three agents, because not all the members can guarantee more profit by forming coalition with others. The reason is as follows. The IC already seizes all the maximum market profit, regardless of the cooperation or competition between the shipper and carrier, thus it has no incentive to cooperate with any other agents. Because all the profit of the market is already extracted by the IC, the shipper and the carrier have no incentive to cooperate either.

**Example 1.** For the one shipper, one carrier and one IC problem, we let

\[
p(x) = 20 - 0.2x, \quad c_1(x) = 5x, \quad c_2(x) = 2x + 0.03x^2, \quad c_3(x) = 3x + 0.02x^2
\]

Maximizing the joint profit of all the three agents results in

\[
x^* = 20, \quad \Gamma(x^*) = 100
\]

\( a = 3.8, \quad b = 92, \quad \bar{a} = 7, \quad \bar{b} = 80 \)

\( R(x) = 80 + 7x \)

\( S(x) = 92 + 3.8x \)

The IC extracts all the profit with \( \pi_1 = 0, \pi_2 = 0 \) and \( \pi_3 = 100 \).

3. Cooperative game among ICs

Now we move to the case with \( n \) ICs, and consider the game between the ICs as an \( n \)-person cooperative game. The economic reason for this consideration is that competition among the ICs will negatively affect each other’s profit. For an extreme situation – the competitive market, each company will receive zero profit under perfect competition. So cooperation may emerge if permitted. In addition, all ICs within the coalition will choose the same tariff, because, otherwise, the carriers will choose the IC that offers the lowest tariff.

Let \( N = \{1, 2, \ldots, n\} \) be the set of ICs and \( M \) be a subset of \( N \). Let \( \Gamma^*_M \) be the total maximum profit for each subset \( M \) of the ICs, and assume that \( \Gamma^*_M \) is nonnegative and monotonically increasing in \( M \), which implies that

\[
\Gamma^*_M \leq \Gamma^*_{M+i}, \quad \forall i \in N \text{ and } i \notin M
\]

This assumption just corresponds to the subadditivity property of the cost functions (Young, 1994). To allocate the total profit, we consider the following cost-saving game:

**Cost-saving game:** For each IC \( i \), define its marginal savings to be

\[
v_i^* = \Gamma^*_N - \Gamma^*_{N-i}
\]

We use the ACA (alternate cost avoided method) to impute savings according to the formula

\[
\pi_i = \frac{v_i^*}{\sum_{j \in N} v_j^*} v_N
\]

For the game considered here, we have \( v_N = \sum_{j \in N} v_j^* \). Thus \( \pi_i = v_i^* \).

It is reasonable to consider the profit of each IC as the marginal contribution to the joint surplus. This method was proposed in the game theory literature as a means of minimizing players’ “propensity to disrupt” the solution (Young, 1994). After each IC receives its separable profit, the “non-separable profit” will be allo-
cated to the joint surplus of the shipper and the carrier. The economic reason is that when there is more than one IC, the coalition of carriers will exert its market power in their selection of the IC. Because of the competition among the carriers, the total producer surplus can also be partially extracted by the shipper.

**Lemma 3.** Let \( C_{3,i}(x_{3,i}) \) be the \( i \)-th IC's cost function, \( \mu_{3,i}(x_{3,i}) \) be the \( i \)-th IC's producer surplus. Suppose that the coalition of \( n \) ICs offers a two-part tariff of the form: \( S(x) = b + ax \) for \( x > 0 \), where

\[
\begin{align*}
  a &= a^* = MC_{3,1}(x_{3,1}^*) = MC_{3,2}(x_{3,2}^*) = \cdots = MC_{3,n}(x_{3,n}^*) \\
  b &= b^* = \pi_3 - \sum_{i=1}^{n} \mu_{3,i}(x_{3,i}^*)
\end{align*}
\]

The strategy \((a^*, b^*)\) and \( x^* \) is an equilibrium, with profits

\[
\begin{align*}
  \pi_{3,i} &= \Gamma_{N}^* - \Gamma_{N-i}^*, \quad \pi_3 = \sum_{i=1}^{n} \pi_{3,i}, \quad \pi_1 + \pi_2 = \Gamma_{N}^* - \pi_3
\end{align*}
\]

**Proof.** We know that no matter whether the carrier selects an IC \( k \), IC \( k \) is still in the coalition of ICs. Here if we regard the ACA method as a contract between the carrier and the coalition of ICs, then the other ICs' profits will still be \( \pi_{3,i} = \Gamma_{N}^* - \Gamma_{N-i}^* \), \( i \neq k \). Suppose that the carrier chooses the coalition of ICs except IC \( k \). Then the maximum total profit of the shipper and carrier that can be realized is

\[
\pi_1^* + \pi_2^* = \Gamma_{N-k}^* - (\pi_3 - \pi_{3,k}) = \Gamma_{N}^* - \pi_3 \leq \pi_1 + \pi_2
\]

Thus, there is no incentive for the carrier not to select the coalition, and by Lemma 1 the shipper will choose to produce an output \( x^* \). On the other hand, if IC \( k \) leaves the coalition and the carrier selects IC \( k \) to move the shipment, the maximum profit that IC \( k \) could realize is:

\[
\pi_{3,k}^* = \Gamma_{k}^* - \Gamma_{N-k}^* \leq \Gamma_{N}^* - \Gamma_{N-k}^* = \pi_{3,k}
\]

Therefore, there is no incentive for any of the ICs to leave the coalition. This completes the proof. \( \square \)

### 4. A network of carriers and ICs

When there is more than one carrier, for the same reason, the shipper can extract part of the surplus from the coalition of carriers. Thus, the joint surplus will be re-allocated among all of the three kinds of agents.

Let \( X_1 \) be the vector of network flows when the shipper sends (through \( m \) carriers) \( q \) to the market, and \( X_2 \) be the vector of network flows when the carriers send (through \( n \) ICs) \( q \) to the market. The transportation costs for the carriers and ICs are \( c_2(X_1) \) and \( c_3(X_2) \), and the joint system profit is

\[
\Gamma(q, X_1, X_2) = \eta p(q) - c_1(q) - c_2(X_1) - c_3(X_2)
\]

We denote \( q^* \), \( X_1^* \), \( X_2^* \) the vertically efficient flows with profits \( \Gamma^* \), and MC\(_2(X_1^*) \) and MC\(_3(X_2^*) \) the marginal costs along paths with positive flows.

**Proposition 2.** Suppose that the coalition of \( n \) ICs offers a two-part tariff of the form: \( S(x) = b + ax \) and the coalition of \( m \) carriers offers a two-part tariff of the form: \( R(x) = \bar{b} + \bar{a}x \), for \( x > 0 \), where

\[
\begin{align*}
  a &= MC_2(X_1^*), \quad \bar{a} = MC_3(X_2^*) \\
  b &= \pi_3 - \sum_{i=1}^{n} \mu_{3,i}(x_{3,i}^*) \\
  \bar{b} &= \pi_2 - \sum_{i=1}^{n} \mu_{2,i}(x_{2,i}^*) - \sum_{i=1}^{n} \mu_{3,i}(x_{3,i}^*)
\end{align*}
\]
Furthermore, we define $\Gamma^{*}_{-j}$ as the maximum profit without agent $j$, then the strategy $(a, b, \bar{a}, \bar{b})$ and $x^{*}$ is an equilibrium for game of the one shipper, $m$ carriers and $n$ ICs, with

$$
\pi_{1} = \Gamma^{*} - \pi_{2} - \pi_{3} \tag{41}
$$

$$
\pi_{2,j} = \Gamma^{*} - \Gamma^{*}_{-j}, \quad \pi_{2} = \sum_{j=1}^{m} \pi_{2,j} \tag{42}
$$

$$
\pi_{3,i} = \Gamma^{*} - \Gamma^{*}_{-i}, \quad \pi_{3} = \sum_{i=1}^{n} \pi_{3,i} \tag{43}
$$

Proof. The proof is similar to that of Lemma 3. \qedsymbol

To show the existence of the resulting equilibrium, we provide a simple numerical example:

Example 2. For the one shipper, three carriers and two ICs, let

Shipper: $p(x) = 20 - 0.2x$, $c_1(x) = 5x$

Carriers: $c_{2a}(x_{2a}) = 4x_{2a} + 0.05x_{2a}^2$, $c_{2b}(x_{2b}) = 4.5x_{2b} + 0.05x_{2b}^2$, $c_{2c}(x_{2c}) = 5x_{2c} + 0.05x_{2c}^2$

ICs: $c_{3a}(x_{3a}) = 3x_{3a} + 0.02x_{3a}^2$, $c_{3b}(x_{3b}) = 3.5x_{3b} + 0.02x_{3b}^2$

Then,

$$
\Gamma(q, X_1, X_2) = qp(q) - c_1(q) - c_2(X_1) - c_3(X_2)
$$

$q^* = 15.99$, $X_1^* = [10.33, 5.33, 0.33]$, $X_2^* = [14.24, 1.75]$, $\Gamma(x^*) = 62.04$

$R(x) = 89.11 + 5.03x$

$S(x) = 66.95 + 3.57x$

with $\pi_1 = 43.56$, $\pi_2 = 9.86$ and $\pi_3 = 8.62$.

We conclude this section by providing a few remarks. The asymmetry of the information held by the players is an intrinsic factor that leads to the above result. The carrier and the IC need to know the costs of their competitors and the demand and cost functions for the shippers. The IC also needs to know the cost functions of the carriers. Shippers only make decision by their own profit functions and know nothing about carriers and ICs. The information requirements for agents here may be a little stringent. But, the higher level agents do not really need to have all the information mentioned above. By introducing dynamic tariffs, higher level agents can continuously change the tariff a little bit and get the feedback from the system. Finally, the higher level agents can capture the shape of the demand and the tariff will converge to the equilibrium solution of the model. To further illustrate this we provide a possible method here. Suppose in a one shipper, one carrier and one IC game, the IC does not have any information about the shipper and the carrier. However, it still can observe a relationship between its own profit and the tariff it provides, $\pi(a, b)$, where $a, b$ are the two parameters in the nonlinear tariff function. By changing the tariff slightly, it is not hard to obtain the gradient, $\nabla \pi(a, b)$, and the reciprocal of the second derivative with the inverse of the Hessian matrix, $\nabla^2 \pi(a, b)$. Utilizing the Newton’s method, the IC obtains the iterative scheme

$$
(a_{n+1}, b_{n+1}) = (a_n, b_n) - [\nabla^2 \pi(a_n, b_n)]^{-1}\nabla \pi(a_n, b_n), n \geq 0 \tag{44}
$$

As we have assumed, the total profit function of the delivery market is concave. Then it is guaranteed that this method can find the maxima of the function and finally the IC extracts all the surplus of the market. The tariff charged by the IC will converge to the equilibrium solution.

5. Conclusion

We have developed an equilibrium model of competitive freight networks and derived some important properties of the equilibrium solution. The major observations made from this study can be summarized as follows:
1. If the tariffs associated with the shipper, the carrier and the IC can be determined by themselves, the forms of nonlinear tariffs are system-wide preferred. Upper-level agents (the IC and the carrier) can use nonlinear tariffs to induce the lower-level agents (the carrier and the shipper) to move the vertically efficient flows.

2. The equilibrium flows can be easily calculated by solving the integrated system optimization problem. The model predicts the tariff schedules adopted by both the carrier and the IC. The UE and SO can be simultaneously achieved by using the nonlinear tariff.

3. The agents who give the tariff may take great advantage in sharing the surplus, because the first part of the tariff can be determined independently of the production quantity. But the competition among the coalition of the same kind of agents can greatly weaken the market power of this coalition. Higher level agents have the power to decide the tariff, while lower level agents have the power to choose the agents, causing the surplus to be reallocated.

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References